

CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Ordinary Level

ADDITIONAL MATHEMATICS

4037/02

Paper 2

May/June 2003

2 hours

Additional Materials: Answer Booklet/Paper
Graph paper
Mathematical tables

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Write your answers on the separate answer booklet/paper provided.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 80.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

1. ALGEBRA*Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \dots + \binom{n}{r} a^{n-r}b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)! r!}$.

2. TRIGONOMETRY*Identities*

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$\Delta = \frac{1}{2} bc \sin A.$$

- 1 Given that $4x^4 - 12x^3 - b^2x^2 - 7bx - 2$ is exactly divisible by $2x + b$,
- (i) show that $3b^3 + 7b^2 - 4 = 0$,
- (ii) find the possible values of b . [5]
- 2 The position vectors of points A and B , relative to an origin O , are $6\mathbf{i} - 3\mathbf{j}$ and $15\mathbf{i} + 9\mathbf{j}$ respectively.
- (i) Find the unit vector parallel to \overrightarrow{AB} . [3]
- The point C lies on AB such that $\overrightarrow{AC} = 2\overrightarrow{CB}$.
- (ii) Find the position vector of C . [3]
- 3 Express $\int_1^8 \left(3\sqrt{x} + \frac{2}{\sqrt{x}} \right) dx$ in the form $a + b\sqrt{2}$, where a and b are integers. [6]
- 4 Simplify $\frac{16^{x+1} + 20(4^{2x})}{2^{x-3}8^{x+2}}$. [4]
- 5 A function f is defined by $f: x \mapsto \frac{e^x + 1}{4}$ for the domain $x \geq 0$.
- (i) Evaluate $f^2(0)$. [3]
- (ii) Obtain an expression for f^{-1} . [2]
- (iii) State the domain and the range of f^{-1} . [2]
- 6 Find the solution set of the quadratic inequality
- (i) $x^2 - 8x + 12 > 0$, [3]
- (ii) $x^2 - 8x < 0$. [2]
- Hence find the solution set of the inequality $|x^2 - 8x + 6| < 6$. [2]

- 7 (i) Find the number of different arrangements of the letters of the word MEXICO.

Find the number of these arrangements

(ii) which begin with M,

(iii) which have the letter X at one end and the letter C at the other end.

[5]

Four of the letters of the word MEXICO are selected at random. Find the number of different combinations if

(iv) there is no restriction on the letters selected,

(v) the letter M must be selected.

[3]

- 8 (a) Find all the angles between 0° and 360° which satisfy the equation

$$3(\sin x - \cos x) = 2(\sin x + \cos x).$$

[4]

(b) Find all the angles between 0 and 3 radians which satisfy the equation

$$1 + 3 \cos^2 y = 4 \sin y.$$

[4]

- 9 A motorcyclist travels on a straight road so that, t seconds after leaving a fixed point, his velocity, $v \text{ ms}^{-1}$, is given by $v = 12t - t^2$. On reaching his maximum speed at $t = 6$, the motorcyclist continues at this speed for another 6 seconds and then comes to rest with a constant deceleration of 4 ms^{-2} .

(i) Find the total distance travelled.

[6]

(ii) Sketch the velocity-time graph for the whole of the motion.

[2]

- 10 A curve has the equation $y = \frac{2x + 4}{x - 2}$.

(i) Find the value of k for which $\frac{dy}{dx} = \frac{k}{(x - 2)^2}$.

[2]

(ii) Find the equation of the normal to the curve at the point where the curve crosses the x -axis. [4]

A point (x, y) moves along the curve in such a way that the x -coordinate of the point is increasing at a constant rate of 0.05 units per second.

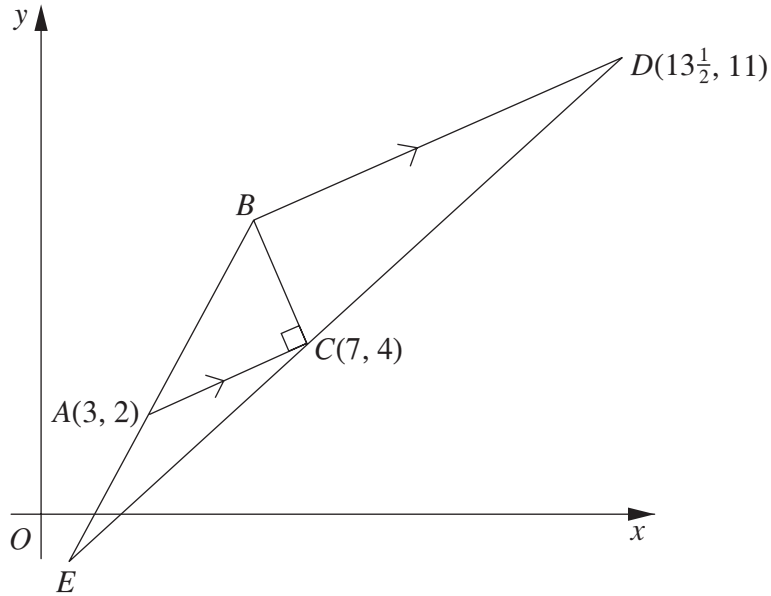
(iii) Find the corresponding rate of change of the y -coordinate at the instant that $y = 6$.

[3]

11 Answer only **one** of the following two alternatives.

EITHER

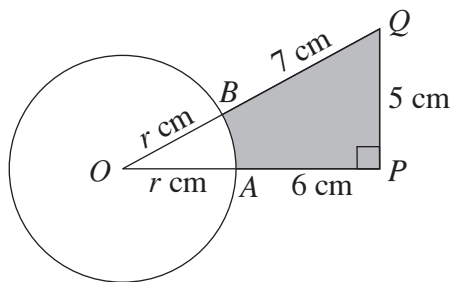
Solutions to this question by accurate drawing will not be accepted.



The diagram shows a triangle ABC in which A is the point $(3, 2)$, C is the point $(7, 4)$ and angle $ACB = 90^\circ$. The line BD is parallel to AC and D is the point $(13\frac{1}{2}, 11)$. The lines BA and DC are extended to meet at E . Find

- (i) the coordinates of B , [7]
 (ii) the ratio of the area of the quadrilateral $ABDC$ to the area of the triangle EBD . [3]

OR



The diagram shows a right-angled triangle OPQ and a circle, centre O and radius r cm, which cuts OP and OQ at A and B respectively. Given that $AP = 6$ cm, $PQ = 5$ cm, $QB = 7$ cm and angle $OPQ = 90^\circ$, find

- (i) the length of the arc AB , [6]
 (ii) the area of the shaded region. [4]

